$\pi\pi$ and $\pi^- p \rightarrow \pi^0 n$ Scattering in a Multiple-Reggeon-Cut Dual Absorptive Model

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A dual absorptive amplitude, in which s-channel resonances are generated by summing up t-channel n-Reggeon cuts, is constructed. Adjusting the two free parameters of the model in the resonance region of $\pi\pi$ scattering, the various $\pi\pi$ differential high energy cross sections are calculated. The model is also applied to πN charge exchange, where a fit to the high energy data is obtained even without Reggeon-Pomeron cuts.

I. Introduction

In the last years there have been many attempts to remove the difficulties of Regge pole models by introducing absorptive contributions in the form of Regge cuts. Recent polarization data of the pion-nucleon charge-exchange reaction [1] have, however, shown strong disagreement with the predictions of the two most popular absorption models, the weak-cut [2] and the strong-cut model [3]. In addition, these models were not able to describe the $t = -0.6 \text{ GeV}^2$ dip structure in the differential cross sections of various inelastic two body processes consistently. In 1971 Harari showed that these difficulties as well as the crossover problem could be solved by imposing duality constraints to the amplitude [4, 5].

In this paper we will present a model of a dual absorptive amplitude, which can be regarded as a realization of Harari's ideas. With a simple two parameter Regge pole term realistic s-channel resonances are produced by summing up t-channel Regge poles and n-Reggeon cuts. If we fit these parameters in the resonance region of pion-pion scattering, we can predict the $\pi\pi$ differential cross sections at high energies (adding a simple phenomenological Pomeron term). The results are in full agreement with the predictions of Harari's dual absorptive model [5].

In the second chapter we construct our model amplitude and investigate its properties in the low and high energy regions.

Chapter III deals with the application of the model to pion-pion scattering. We calculate the elastic and total cross sections and show the dual properties of the nondiffractive part of the amplitude. Finally

we predict the differential cross sections of the various pion-pion scattering processes at high energies.

In Sect. IV the model is applied to the pion-nucleon charge-exchange reaction, and we obtain a good fit to the high energy data including polarization even without Reggeon-Pomeron cuts.

II. Construction of a Dual Absorptive Model Amplitude

In analogy to the Veneziano amplitude [6] in pole-pole duality, where s-channel poles are generated by an infinite sum of t-channel Regge terms

$$V_{\text{venez.}}(s,t) = \frac{\Gamma((1-\alpha(t))\,\Gamma((1-\alpha(s)))}{\Gamma(1-\alpha(t)-\alpha(s))}$$
$$\underset{s\to\infty}{\sim} \Gamma(1-\alpha(t))$$
$$\cdot \sum_{n=0}^{\infty} C_n(t)\,(-\alpha's)^{\alpha(t)-n} \qquad (1)$$

with $\alpha(s) = \alpha_0 + \alpha's$, we want to get s-channel resonances by summing up n-Reggeon cuts (Fig. 1):

$$V(s,t) = R(s,t) + (R \otimes R)(s,t)$$
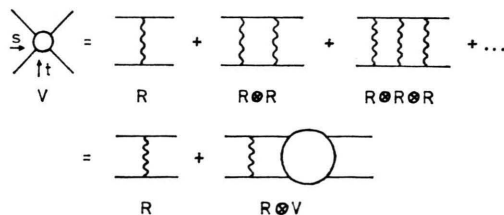$$+ (R \otimes R \otimes R)(s,t) + \dots = \qquad (2)$$
$$= R(s,t) + (R \otimes V)(s,t),$$

Fig. 1. Diagrams illustrating Equation (2).

where $R(s, t)$ is the Regge term and $R \otimes R$ is defined by multiplication of the partial wave projections:

$$\frac{1}{2} \int_{-1}^{1} dz \, (R \otimes R) \, (s, z) \, P_l(z) \qquad (3)$$
$$= (R \otimes R)_l(s) := R_l(s) \, R_l(s) \, .$$

Equation (2) can therefore be written as

$$V_l(s) = R_l(s) / \left(1 - R_l(s)\right) \, . \qquad (4)$$

Obviously there will be resonances with spin l at the places s_i given by

$$R_l(s_i) = 1 \quad \left(R_l'(s_i) \neq \right)) \, . \qquad (5)$$

In order to get an amplitude $V(s, t)$ which has a cut from $4 \, m^2$ to ∞ in the s-plane, we take a Regge term $R(s, t)$ of the following form:

$$R(s, t) = -\beta(t) \left(\frac{4 \, m^2 - s}{s_0} \right)^{a(t)}$$
$$= -\beta(t) e^{-i\pi a} \left(\frac{s - 4 \, m^2}{s_0} \right)^{a(t)} \, . \qquad (6)$$

$\beta(t)$ should have poles at $\alpha(t) = 1, 2, 3 \ldots$:

$$\beta(t) = \tilde{\beta}(t) \, \Gamma\left(1 - \alpha(t)\right) \, . \qquad (7)$$

Because of the integration over t, the terms $R \otimes R$, $R \otimes R \otimes R$, ... do not possess t-channel poles. from Eq. (2), (6), and (7) we get

$$\operatorname*{Res}_{t = t_n} V(s, t) = \qquad (8)$$
$$\operatorname*{Res}_{t = t_n} V(s, t) = \frac{-\tilde{\beta}(t_n) \, (-1)^n}{\alpha'(t_n) \, (n-1)!} \left(\frac{4 \, m^2 - s}{s_0} \right)^n \, ,$$

which is a polynomial in $\cos \theta_t$ of degree n.

Primarily we want to describe the s-channel physical region, of course, so that we will confine ourselves from now on to negative t-values, where for the purpose of analytical calculations we make the approximations

$$\alpha(t) \approx \alpha_0 + \alpha' \, t, \quad \beta(t) \approx \beta_0 \; (\text{for } t \leq 0) \, . \qquad (9)$$

Our model is now defined by

$$R(s, t) = -\beta_0 \left(\frac{4 \, m^2 - s}{s_0} \right)^{\alpha_0 + \alpha' t} \qquad (t \leq 0) \quad (10)$$

$$V(s, t) = R(s, t) + (R \otimes V) \, (s, t) \qquad (11)$$
$$\text{or} \quad V_l(s) = R_l(s) / \left(1 - R_l(s)\right) \, .$$

For the partial wave projection of the Regge term (10) we get

$$R_l(s) = -\beta_0 \, e^{-i\pi a_0} \sqrt{\frac{\pi}{2 \, A}} \left(\frac{s - 4 \, m^2}{s_0} \right)^{a_0} e^{-A} I_{l + \frac{1}{2}} (A)$$

$$\qquad (12)$$
with
$$A = \frac{\alpha'}{2} \, (s - 4 \, m') \left(\ln \frac{s - 4 \, m^2}{s_0} - i \, \pi \right) \, . \quad (13)$$

$I_{l + \frac{1}{2}}$ is the modified Bessel function and has the expansion

$$I_{l + \frac{1}{2}} (A) = \sqrt{\frac{1}{2} A} \, (\tfrac{1}{2} A)^l \sum_{k=0}^{\infty} \frac{(\tfrac{1}{2} A)^{2k}}{k! \, \Gamma(l + k + \tfrac{3}{2})} \, . \quad (14)$$

I_l fulfills the conditions of Carlson's theorem [7], so that (12) with (14) is the desired analytical continuation of $R_l(s)$ into the complex l-plane, Obviously $V_l(s)$ can also be analytically continued,

$$V(l, s) = R(l, s) / \left(1 - R(l, s)\right) \, , \qquad (15)$$

and the s-channel Regge trajectories are given by

$$R(l, s) \big|_{l = a(s)} = 1 \, . \qquad (16)$$

Here it is not necessary to introduce signatured amplitudes, because $V(s, t)$ has no u-channel singularities.

The numerical calculation of $\alpha(s)$ for $0.5 \, \text{GeV}^2 \leq s \leq 2.5 \, \text{GeV}^2$ with the input trajectory $\alpha(t) = 0.5 + t$ and several values of β_0 and s_0 shows surprisingly good output trajectories $\operatorname{Re} \alpha(s) \approx \alpha_0 + \alpha' s$ with $\alpha' \approx 1.2 \, \text{GeV}^{-2}$ and α_0 depending on β_0 and s_0 (as an example see Figure 2). The poles are placed on the second sheet below the s-plane cut as desired. $\operatorname{Im} \alpha(s)$ has positive values in the right order of magnitude and increases with s.

Turning to the high energy region, the formula

$$\frac{1}{\pi \, s} \int_{-\infty}^{0} dt_1 \int_{-\infty}^{0} dt_2 \, \frac{\Theta(\tau)}{\sqrt{\tau}} \, e^{A t_1} \, e^{B t_2} = \frac{\exp\left(\dfrac{A \, B}{A + B} \, t \right)}{s \, (A + B)}$$

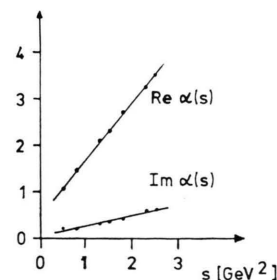with $\quad \tau := -t^2 - t_1^2 - t_2^2 + 2 \, (t \, t_1 + t \, t_2 + t_1 \, t_2)$

Fig. 2. Some calculated points of the output trajectory $\alpha(s)$ obtained with the input trajectory $\alpha(t) = 0.5 + t$, $\beta_0 = 4$, and $s_0 = 1 \, \text{GeV}^2$. The real part is approximately given by $\operatorname{Re} \alpha(s) = 0.45 + 1.2 \, s$.

allows us to calculate the n-Reggeon terms of expansion (2) for $|t| \ll s$[8]:

$$V(s,t) = -\sum_{n=1}^{\infty} \frac{\beta_0{}^n \, e^{-i\pi a_n(t)} \left(\dfrac{s}{s_0}\right)^{a_n(t)}}{n \, (s_0 \, \alpha')^{n-1} \left(\ln \dfrac{s}{s_0} - i\pi\right)^{n-1}} \quad (17)$$

with

$$a_n(t) = 1 + n(\alpha_0 - 1) + \frac{\alpha'}{n} t. \quad (18)$$

$a_n(t)$ is the trajectory of the branch point of the n-Reggeon cut. Figure 3 shows $a_n(t)$ together with the effective trajectory $\alpha_{\text{eff}}(t)$, which is given by

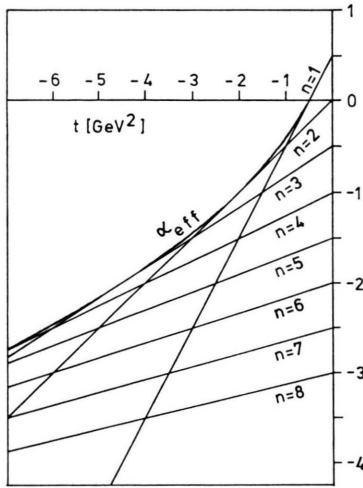$$\alpha_{\text{eff}}(t) = 1 - 2\sqrt{1 - \alpha_0} \ \sqrt{-\alpha' t}. \quad (19)$$

Fig. 3. Plot of the n-Reggeon cut trajectories $\alpha_n(t)$ and their envelope $\alpha_{\text{eff}}(t)$ with $\alpha_1(t) \equiv \alpha(t) = 0.5 + t$.

For the application to $\pi\pi$ or πN scattering we also have to know $V(u,t)$ and $V(s,u)$ at high energies. It is easily seen, that $V(s,u)$ vanishes exponentially for $s \to \infty$ at fixed t and can therefore be neglected. The calculation of $V(u,t) \approx V(-s,t)$ would require an integration over positive values of t, where our approximation $\beta(t) \approx \beta_0$ is no longer valid and would actually lead to divergencies. Instead of postulating a definite functional behaviour of $\beta(t)$ for positive t we assume, that $\beta(t)$ falls of rapidly enough for $t \to \infty$ as to avoid an eponential increase of $V(u,t)$ for $s \to \infty$, and take

$$V(u,t) = -\sum_{n=1}^{\infty} \frac{\beta_0{}^n \left(\dfrac{s}{s}\right)^{a_n(t)}}{n \left(s_0 \, \alpha' \ln \dfrac{s}{s_0}\right)^{n-1}} \quad \left(\frac{|t|}{s} \ll 1\right), \quad (20)$$

which seems to be quite reasonable.

Finally we have to incorporate the Pomeron. If we make the simple phenomenological ansatz

$$P(s,t) = i \, s \, \sigma \, e^{ct} \quad (21)$$

for the diffractive part of the amplitude, an iteration of this term like Eq. (2) (sum of n-Pomeron cuts) would only lead to a renormalization of the residue function. So we will regard the expression (20) as the result of such a summation, and only cuts of one Pomeron and n Reggeons have to be added to the amplitude:

$$\tilde{V}(s,t) = V(s,t) + 2i(V \otimes P)(s,t) \quad (22)$$

$$A(s,t) = P(s,t) + g \, \tilde{V}(s,t) \quad (g = \text{const}) \quad (23)$$

or with (4):

$$A_l(s) = P_l(s) + g \, \tilde{V}_l(s) = P_l(s) + \frac{g \, R_l(s)}{1 - R_l(s)} \cdot (1 + 2i P_l(s)). \quad (24)$$

Figure 4 shows the various pole and cut terms in $A(s,t)$. The additional absorption by Reggeon-Pomeron cuts in $V_l(s)$ of course does not affect the s-channel pole structure of our amplitude, as is seen from Equation (24).
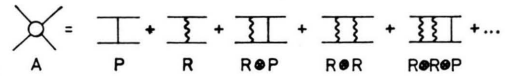

Fig. 4. Diagrams illustrating the ansatz (24).

III. Application to Pion-Pion Scattering

Isospin invariance and crossing symmetry reduces the number of independent pion-pion amplitudes to one. If we take this to be $A(\pi^+\pi^- \to \pi^+\pi^-) \equiv F(s,t)$ we get

$$\begin{aligned}
A^{I=2}(s,t) &= F(u,t), \\
A^{I=1}(s,t) &= F(s,t) - F(s,u), \\
A^{I=0}(s,t) &= \tfrac{3}{2}\big(F(s,t) + F(s,u)\big) - \tfrac{1}{2} F(u,t).
\end{aligned} \quad (25)$$

We will normalize the amplitudes in such a way that the Optical Theorem reads

$$\sigma_T^I = 32 \, \pi / \big(s(s - 4 m^2)\big) \quad \text{Im} \, A^I(s, t=0). \quad (26)$$

The elastic and the differential cross sections are then given by

$$\sigma_{\text{el}}^I = \frac{32 \, \pi}{s} \sum_{l=1}^{\infty} (2l+1) |A_l^I(s)|^2 \quad (27)$$

and

$$\frac{d\sigma^I}{dt} = \frac{64 \, \pi}{s(s - 4 m^2)} |A^I(s,t)|^2. \quad (28)$$

Now we express $F(s, t)$ by our model amplitude:

$$F(s, t) = P(s, t) + g \, \tilde{V}(s, t), \qquad (29)$$

$$\tilde{V}_l(s) = \frac{R_l(s)}{1 - R_l(s)} \left(1 - 2 \, i \, P_l(s)\right) \qquad (30)$$

with

and

$$P(s, t) = \text{is} \; \sigma \; \mathrm{e}^{ct} \qquad (31)$$

$$R(s, t) = -\beta_0 \left(\frac{4 \, m^2 - s}{s_0}\right)^{a_0 + a't} \quad (t \lessgtr 0), \quad (32)$$

where ϱ- and f-trajectories are taken to be exchange degenerate:

$$\alpha_\varrho(t) = \alpha_f(t) = 0.5 + 0.9 \, t. \qquad (33)$$

As there are no data on high energy $\pi \pi$ scattering, we have to fix the Pomeron parameters arbitrarily, say *

$$\sigma = 0.5 \; \text{GeV}^{-2}, \quad c = 3 \; \text{GeV}^{-2}, \qquad (34)$$

the value of σ corresponding to an asymptotic $\pi \pi$ total cross section of about 20 mb. At low energies we remove the Pomeron contribution by tetting σ decrease to zero between $s = 1.6 \; \text{GeV}^2$ and $s = 0.8 \; \text{GeV}^2$.

There remain three free parameters: g, β_0, and s_0. In order to get the ϱ-, f-, and g-meson to the right place, we take according to Fig. 4 the values

$$\beta_0 = 4, \quad s_0 = 1 \; \text{GeV}^2. \qquad (35)$$

Finally we fix the parameter g by equating the $I = 1$ elastic and total cross section at $s = m_\varrho{}^2$ and get

$$g = 0.34. \qquad (36)$$

Figure 5 and Fig. 6 show the resulting elastic cross sections for both the $I = 1$ channel and $\pi^+ \pi^-$ scattering. The resonance structure looks quite realistic. Below $M(\pi \pi) = 1.5 \; \text{GeV}$ there are regions in which the total cross section is smaller than the elastic one indicating a violation of unitarity. To remedy this, the Reggeterm $R(s, t)$ should have a much more complicated s-dependence at low energies than $(s - 4 \, m^2)^{a(t)}$, which is of course too crude. On the other hand a reggeized K-matrix ansatz, which is explicitly unitary, would comprehend the whole amplitude (inculding u-channel singularities) and thus produce difficulties with duality and crossing symmetry.

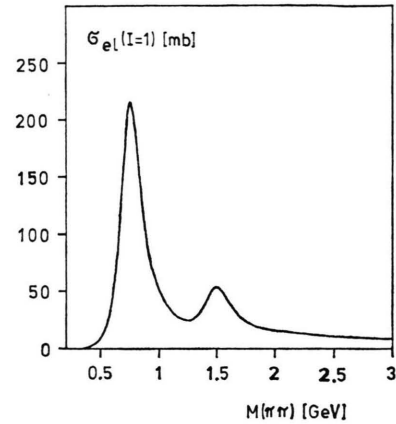* All the qualitative features of our results are insensitive to a change of these parameters.

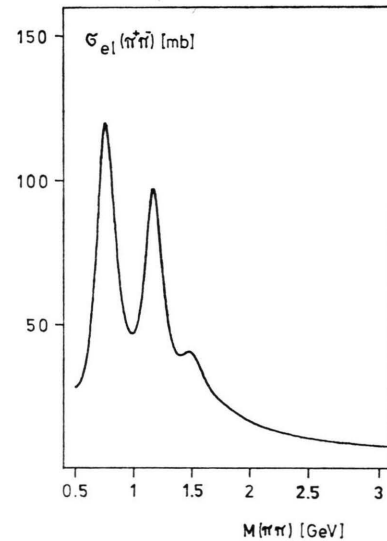Fig. 5. Elastic cross section for the $I = 1$ channel of $\pi \pi$ scattering.

Fig. 6. Elastic cross section for $\pi^+ \pi^-$ scattering.

As the amplitude is now completely fixed, we can compare it with Harari's dual absorptive model [5] and calculate the $\pi \pi$ differential cross sections at high energies. The essential statement of Harari's DAM is that as a consequence of two component duality [9] the imaginary part of the non-diffractive part of the amplitude should be dominated by the most peripheral partial waves, and therefore

$$\text{Im} \, A_{\Delta \lambda}^{\text{nondiffr.}} \sim J_{\Delta \lambda}(r \sqrt{-t}) \quad (r \approx 1 \; \text{fm}) \quad (37)$$

for helicity flip $\Delta \lambda$. Indeed $\text{Im} \, \tilde{V}(s, t)$ behaves like $J_0(r \sqrt{-t})$ with $r \approx 0.8 \; \text{fm}$ (Figure 7). In the impact parameter representation the peripheral
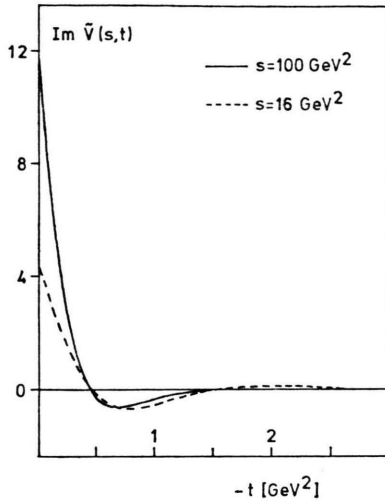
Fig. 7. The non-pomeron imaginary part of the $\pi^+ \pi^-$ amplitude, Im $\tilde{V}(s, t)$, versus t at $s = 16$ and 100 GeV².
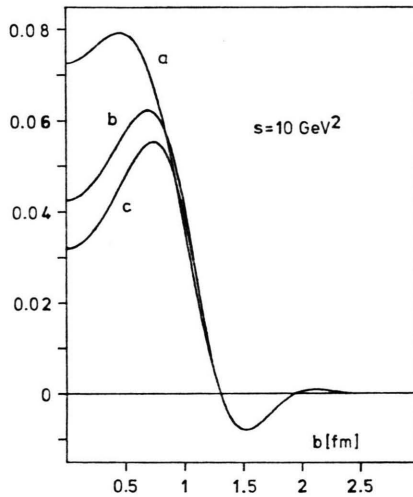

Fig. 8. The impact parameter representation of the imaginary parts of (a) the Reggeterm R, (b) the amplitude V which has absorption by n-Reggeon cuts, and (c) the amplitude $\tilde{V}$ with additional absorption by n-Reggeon-Pomeron cuts, at $s = 10$ GeV².

nature of Im $\tilde{V}$ is clearly seen (Figure 8). It is also seen, that this comes about mainly by absorption of the lower partial waves of the Regge term R by the n-Reggeon cuts contained in V, whereas the additional absorption by Reggeon-Pomeron cuts leading to $\tilde{V}$ is not so important at $s = 10$ GeV². Only at $s \approx 40$ GeV² the two absorptive contributions become equally important.

Figures $9 - 12$ show the $\pi \pi$ differential cross section predictions at $s = 15$, 30, and 100 GeV². In $\pi^+ \pi^-$ scattering we have

$$\frac{d\sigma}{dt} (\pi^+ \pi^-) \propto \left| P(s, t) + \tilde{V}(s, t) \right|^2 \quad (38)$$

$$\approx \left| P(s, t) \right|^2 + 2 \left| P(s, t) \right| \text{Im} \, \tilde{V}(s, t) ,$$

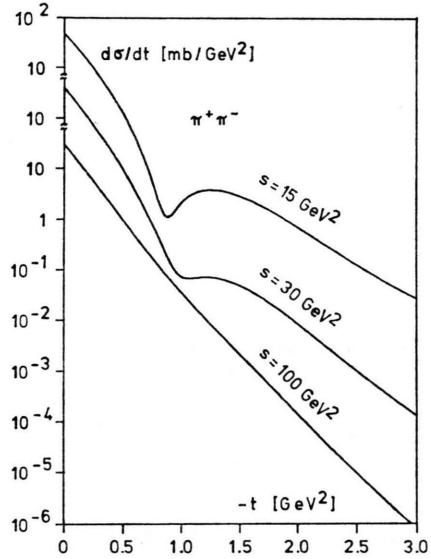so that there will be a dip at the minimum of Im $\tilde{V}(s, t)$, which is filled up with growing energy

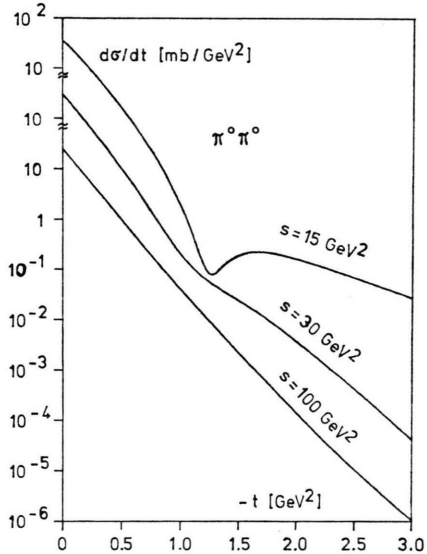Fig. 9. $\pi^+ \pi^-$ differential cross section at $s = 15$, 30, and 100 GeV².

Fig. 10. $\pi^0 \pi^0$ differential cross section at $s = 15$, 30, and 100 GeV².
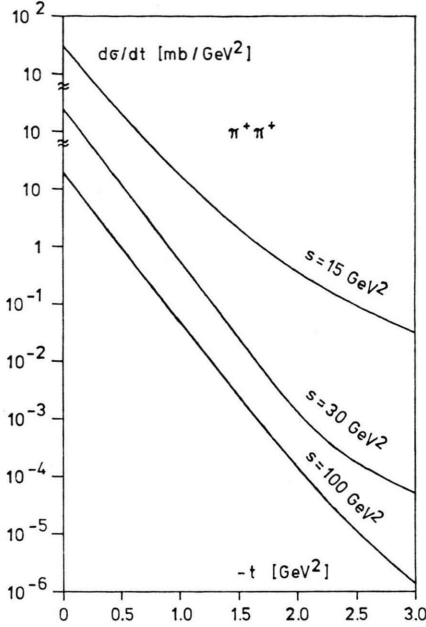
Fig. 11. $\pi^+ \pi^+$ differential cross section at $s=15$, 30, and 100 GeV^2.
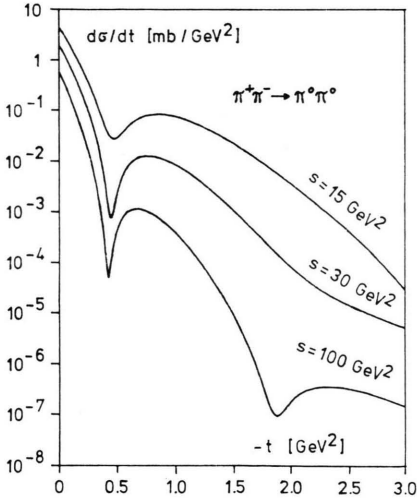

Fig. 12. $\pi\,\pi$ charge exchange differential cross section at $s=15$, 30, and 100 GeV^2.

(Figure 9). As the imaginary part of $\tilde{V}(u, t)$ is zero [see Eq. (20)], the same behaviour is predicted in the $\pi^0 \pi^0$ case (Fig. 10):

$$\frac{d\sigma}{dt} (\pi^0 \pi^0) \propto \left| P(s, t) + \frac{1}{2} \tilde{V}(s, t) + \frac{1}{2} \tilde{V}(u, t) \right|^2$$

$$\approx | P(s, t) |^2 + | P(s, t) | \operatorname{Im} \tilde{V}(s, t). \quad (39)$$

The exotic process $\pi^+ \pi^+ \to \pi^+ \pi^+$ shows no dip structure (Fig. 11), as it is expected from two component duality:

$$\frac{d\sigma}{dt} (\pi^+ \pi^+) \propto | P(s, t) + \tilde{V}(u, t) |^2 \approx | P(s, t) |^2 . \quad (40)$$

Finally for the charge exchange reaction $\pi^+ \pi^- \to \pi^0 \pi^0$ we get

$$\frac{d\sigma}{dt} (\pi^+ \pi^- \to \pi^0 \pi^0) \propto | \tilde{V}(u, t) - \tilde{V}(s, t) |^2 \quad (41)$$

$$\sim | 1 - e^{-i\pi a_{\mathrm{eff}}(t)} |^2 \left(\frac{s}{s_0} \right)^{2a_{\mathrm{eff}}(t)} ,$$

where we have used

$$\tilde{V}(s, t) \approx e^{-i\pi a_{\mathrm{eff}}(t)} \tilde{V}(u, t) \sim e^{-i\pi a_{\mathrm{eff}}} \left(\frac{s}{s_0} \right)^{a_{\mathrm{eff}}} , \quad (42)$$

which is a good approximation at high energy, so that we expect a dip at $t \approx -0.6$ GeV^2 which becomes deeper with increasing energy (Figure 12).

IV. Application to π N Charge-Exchange Scattering

The pion-nucleon charge-exchange data have been fitted by many absorption models of which the most popular ones, the weak-cut [2] and strong-cut model [3], predicted a sharp negative spike of the polarization at $t \approx -0.5$ GeV^2, which clearly disagrees with the new data [1]. Worden [10] has shown that these absorption models are not dual; so we want to test whether imposing duality will remove the difficulties with polarization.

For the sake of simplicity we will apply our model amplitude V to the invariant amplitudes A' and B, which are free of kinematical singularities. The differential cross section of the charge-exchange reaction, the polarization of the recoil nucleon, and the difference between the $\pi^- p$ and $\pi^+ p$ total cross sections are given by

$$\frac{d\sigma}{dt} = \frac{1}{16 \pi p_{\mathrm{L}}^2} \left(\left(1 - \frac{1}{4 M^2} \right) | A' |^2 \right. \quad (43)$$

$$\left. + \frac{t}{4 M^2} \left(s - \frac{(s + M^2 - m^2)^2}{4 M^2 - t} \right) | B |^2 \right),$$

$$P = \left(\frac{d\sigma}{dt} \right)^{-1} \frac{\sin \Theta}{16 \pi \sqrt{s}} \operatorname{Im}(A' B^*), \quad (44)$$

and

$$\Delta\sigma = - \frac{\sqrt{2}}{p_{\mathrm{L}}} \operatorname{Im} A'(s, t=0), \quad (45)$$

where p_L is the pion lab momentum, θ is the c.m. scattering angle, M and m are the nucleon and pion mass, respectively.

The su-crossing symmetry properties of A' and B can be expressed by

$$A'(s,t) = \frac{1}{\sqrt{2}}\left(F(s,t) - F(u,t)\right), \qquad (46)$$

$$B(s,t) = \frac{1}{\sqrt{2}}\left(G(s,t) + G(u,t)\right). \qquad (47)$$

As in the $\pi\pi$ case we will take exchange degenerate ϱ- and f-trajectories

$$\alpha_\varrho(t) = \alpha_f(t) = \alpha_0 + (1-\alpha_0)t/m_\varrho^2 \equiv \alpha(t) \quad (48)$$

and make the following ansatz for F and G:

$$F(s,t) = g_0 \, V(s,t; \beta_0, s_0, \alpha_0), \qquad (49)$$

$$G(s,t) = g_1 \frac{s_1}{s} \, V(s,t; \beta_1, s_1, \alpha_0) \qquad (50)$$

where V is defined by

$$V_l(s) = R_l(s)/[1 - R_l(s)] \qquad (51)$$

with

$$R(s,t; \beta_i, s_i, \alpha_0) = -\beta_i \left(\frac{(M+m)^2 - s}{s_i}\right)^{\alpha(t)} \qquad (52)$$

$$\cdot \, (i = 0, 1; \; t \leq 0),$$

The model amplitude $V(s,t)$ contains n-Reggeon cuts only; but even without Reggeon-Pomeron cut contributions we have got a good fit to the following data (see Figs. 13, 14, and 15):

(a) Difference between $\sigma_t(\pi^- p)$ and $\sigma_t(\pi^+ p)$ for $4.83 \leq p_L \leq 60 \, \text{GeV}$ (data from Refs. 11, 12, 13).

(b) Differential cross section for $0 \leq -t \leq 2 \, \text{GeV}^2$ and $4.83 \leq p_L \leq 18.2 \, \text{GeV}$ (data from Refs. 14, 15, 16).

(c) Polarization for $0 \leq -t \leq 2 \, \text{GeV}^2$ at $p_L = 4.9 \, \text{GeV}$ and $p_L = 7.85 \, \text{GeV}$ (data from Ref. 1).

The χ^2 value of our fit is 1.6 per data point with the following parameter values:

$$g_0 = -1.85 \, \text{GeV}^{-1}, \quad \beta_0 = 0.78, \quad s_0 = 0.02 \, \text{GeV}^2,$$
$$g_1 = 239 \quad \text{GeV}^{-2}, \quad \beta_1 = 1.2, \quad s_0 = 0.22 \, \text{GeV}^2,$$

and

$$\alpha_0 = 0.53.$$

It is also possible to get a good fit with $s_0 \approx 0.2$ GeV^2; the polarization would then become a bit worse, but still without showing a negative spike at $t \approx -0.5 \, \text{GeV}^2$.

We have also calculated the $I_t = 1$ helicity amplitudes at $p_L = 6 \, \text{GeV}$. They are in good agreement with the data of the model independent analysis carried out by Halzen and Michael [17]. Although $\text{Im} \, A'$ and $\text{Im} \, B$ have their first zero at $t \approx -0.6$ GeV^2, $\text{Im} \, F_{++}$ goes through zero at $t \approx -0.25$ GeV^2. This must be due to a compensation between the contributions of A' and B, whereas the strong cut model needs very strong absorption of F_{++} in order to put this zero to the right place.

V. Discussion

Starting from the idea that the s-channel resonances are dual to the sum of t-channel Regge poles and cuts, we have constructed a model amplitude which generates resonances by summing up n-Reggeon cuts. Even a very crude two parameter ansatz
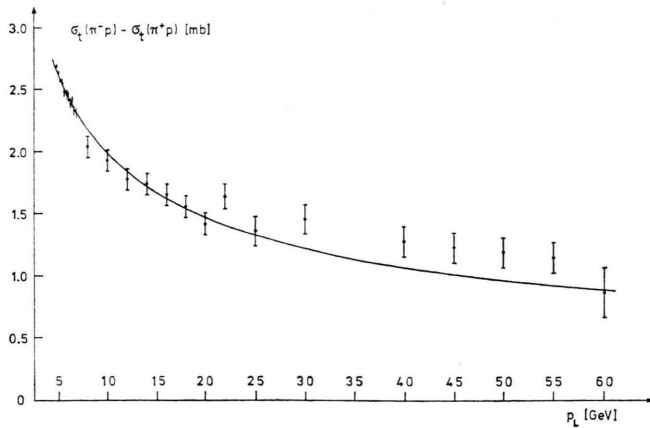

Fig. 13. Fit to the total cross section difference $\sigma_t(\pi^- p) - \sigma_t(\pi^+ p)$ versus laboratory momentum.
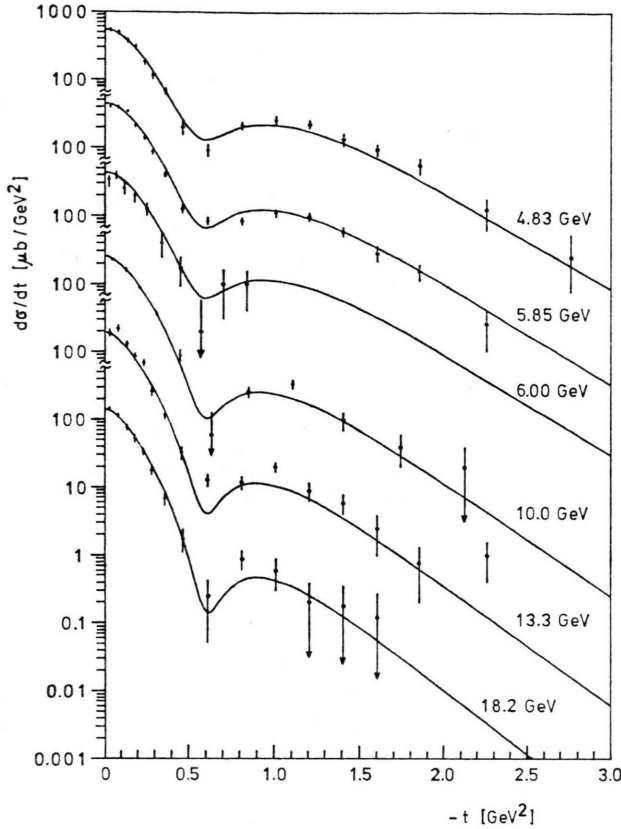
Fig. 14. Fit to the $\pi\,N$ charge exchange differential cross sections.

Fig. 15. Fit to the new polarization data for $\pi^- \mathrm{p} \to \pi^0\,\mathrm{n}$.

for the Regge pole term leads to realistic s-channel output trajectories $\alpha(s)$ with the right sign and magnitude of Im $\alpha(s)$.

In the application to pion-pion scattering the two parameters were fixed by adjusting the resonance peaks of the elastic $\pi^+ \pi^-$ cross section. Using a Pomeron term of the usual form we could predict the high energy $\pi\,\pi$ differential cross sections. Their dip structure as well as the dominance of the peripheral partial waves in the imaginary part of the nondiffractive amplitude correspond completely
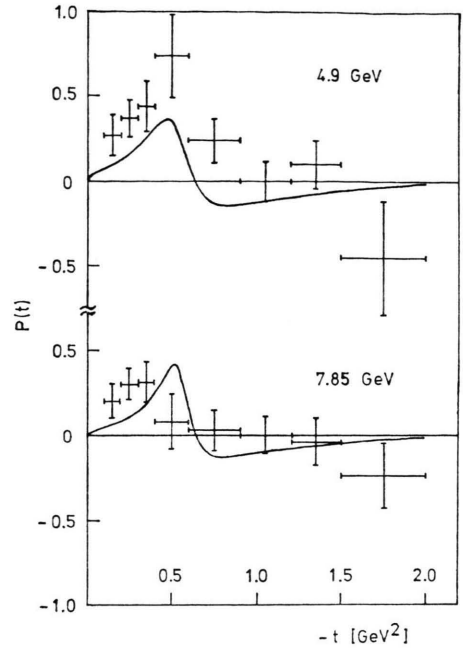
to the conclusions of Harari's dual absorptive model. Up to $s \approx 40\ \mathrm{GeV^2}$ the absorption by n-Reggeon cuts was revealed to be more important than the additional absorption by Reggeon-Pomeron cuts, and the sum ob absorptive contributions produced a peripheral peak in the impact parameter representation at $b = r$ with $r \approx 0.8$ fm.

Our model was also successfully used to fit the high energy data of the pion-nucleon charge-exchange reaction without producing a negative spike in polarization. Of course the fit could have been improved by adding Reggeon-Pomeron cuts or by a more complicated parametrization of the residue function $\beta(t)$. If an appropriate way is found to incorporate the u-channel trajectories N and Δ, it seems to be possible that the fit can be extended to the resonance region of the difference between the $\pi^-\mathrm{p}$ and $\pi^+\mathrm{p}$ total cross sections.

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